

Name of the Course	: <b>Generic Elective</b>
Unique Paper Code	: <b>32355301</b>
Name of the Paper	: <b>GE-3 Differential Equations</b>
Semester	: <b>III</b>
Duration	: <b>3 Hours</b>
Maximum Marks	: <b>75 Marks</b>

*Attempt any four questions. All questions carry equal marks.*

1. (i) Solve

$$x dy - y dx = \sqrt{x^2 + y^2} dx.$$

(ii) Solve the initial value problem

$$(x^2 + y^2 + x) dx + xy dy = 0, \quad y(1) = 1.$$

(iii) Solve the initial value problem

$$x \frac{dy}{dx} + y = y^2 \log x, \quad y(1) = -1.$$

2. (i) Find the orthogonal trajectories of the family of curves  $3xy = x^3 - a^3$ ,  $a$  being parameter of the family.

(ii) Find a family of oblique trajectories that intersect the family of circles  $x^2 + y^2 = c^2$  at angle  $45^\circ$ .

(iii) Solve

$$\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y}.$$

3. (i) Solve by method of variation of parameters

$$y'' + y = \operatorname{cosec} x.$$

(ii) Solve by method of undetermined coefficients

$$y'' + 1.44y = 24 \cos 1.2 x.$$

(iii) Solve

$$y''' - 2y'' + 4y' - 8y = 0, \quad y(0) = -1, \quad y'(0) = 30, \quad y''(0) = 28.$$

4. (i) Show that  $\{e^{-x}, e^{3x}, e^{4x}\}$  forms a basis of the solution set of the equation

$$\frac{d^3 y}{dx^3} - 6 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 12y = 0.$$

- (ii) Solve the initial value problem

$$x^2 y'' - 2x y' - 10y = 0, \quad y(1) = 5, \quad y'(1) = 4.$$

- (iii) Solve the linear system

$$y_1' = 2y_1 + 5y_2$$

$$y_2' = 5y_1 + 12.5y_2$$

5. (i) Find the partial differential equation arising from the surface

$$z = xy + f(x^2 + y^2).$$

- (ii) Find the general solution of the partial differential equation

$$u_x + 2xy^2 u_y = 0.$$

- (iii) Apply the method of separation of variables  $u(x, y) = f(x)g(y)$  to solve

$$y u_x + x u_y = 0 \quad \text{on} \quad u(0, y) = y^2.$$

6. Reduce each of the following equations into canonical form and find the general solution:

(i)  $u_x - u_y = u, \quad u(x, 0) = 4e^{-3x}.$

(ii)  $u_{xx} + 6u_{xy} + 9u_{yy} + 3yu_y = 0.$

(iii)  $u_{xx} - 3u_{xy} + 2u_{yy} = 0.$